A Mathematical Programming Approach to Optimal Design of Dutch Auctions

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Abstract: This paper is concerned with optimal design of Dutch auctions in which the bid levels are restricted to a finite set of values. We formulate this auctioning problem as a constrained nonlinear program and solve it with an aim to maximize the auctioneer's average revenue. Additionally, some important properties of the optimal bid levels as well as the maximum expected revenue are presented.

Keywords: Auction design; Dutch auction; Discrete bids.

I. Introduction

Today, e-commerce is the fastest growing segment in the global economy with e-procurement being one of the major components. E-procurement encompasses several tools with electronic capabilities used to replace the conventional purchasing process, including online catalogs, enterprise resource planning systems, electronic marketplaces, reverse auctions, and Dutch auctions among others [6]. The implementation of e-procurement promises to lower product prices as well as enhance purchasing efficiencies by automating the traditional labor-intensive approach to managing supplies.

II. Existing Works

In a Dutch auction, the auctioneer starts with an extremely high asking price and continues to lower it until a bidder indicates a willingness to buy the object for sale. Competitive bidding processes of this type have been used to conduct a huge volume of business transactions in both the private and the public sectors of the U.S. economy.

Despite their widespread practical applications, there exists only a small body of research on the design of Dutch auctions in general and those with discrete bids in particular [4]. In this paper, we seek to design Dutch auctions to maximize the auctioneer's expected revenue by determining the optimal discrete bid levels. While similar problems have been studied by several researchers [2] [5] [7], our work differs from them in the underlying assumptions and the solution method. For example, it is assumed in this paper that the bidders are risk- neutral and their valuations are independent and private. Besides, we take a mathematical programming approach to solving the Dutch auction problem rather than using a numerical analysis technique.

III. Model Development

Suppose that an object is to be sold in a Dutch auction with $n \geq 1$ bidders and $m \geq 1$ bid levels $l_1 < l_2 < \Lambda < l_m$. The auctioneer begins with an extraordinarily high price and continues to lower it in the sequence of $l_m, l_{m-1}, \mathrm{K}, l_2, l_1$ until a bidder is willing to accept the current asking price. Each bidder's valuation v_j is a uniformly distributed random variable on the support $[0, \bar{v}]$ with PDF $f(.)$ and CDF $F(.)$, j = 1, 2,…, n.

Let Z be the expected revenue to be received by the auctioneer at the end of the Dutch auction and $P(l_i)$ be the probability that the object is sold at the bid level l_i, i = 1, 2, …, m. It follows that

$$P(l_i) = F(l_{i+1})^n - F(l_i)^n$$

$$Z = \sum_{i=1}^{m} l_i P(l_i) = \sum_{i=1}^{m} l_i \left[F(l_{i+1})^n - F(l_i)^n \right]$$

One can show that the Dutch auction model with discrete bid levels may be formulated as follows:

$$\text{Maximize} \qquad Z = \sum_{i=1}^{m} \frac{l_i}{\bar{v}^n} \left(l_{i+1}^n - l_i^n \right)$$

$$\text{subject to:} \qquad l_{i+1} \geq l_i, \; i = 1, 2, \mathrm{K}, \mathrm{m}$$

$$l_1 = 0$$

$$l_{m+1} = \bar{v}$$

Note that the above NLP may be solved efficiently with the aid of any standard optimization software package.

IV. Major Findings

In this section, we present a number of important properties possessed by the optimal bid levels and the auctioneer's maximum expected revenue.

Proposition 1: $l_{i+1}^* - l_i^* < l_i^* - l_{i-1}^*$ if $n > 1$ and $l_{i+1}^* - l_i^* = l_i^* - l_{i-1}^*$ if $n = 1$, $i = 2, 3, \ldots, m$.

Thus, to maximize the expected revenue, each bid should decrease by the same amount as the asking price goes down from one level to another when only one bidder participates in the auction. However, the bid decrements should be increasing if two or more bidders compete for the object for sale.

Proposition 2: (1) Given m, l_i^* is an increasing function of n, $i = 2, 3, \ldots, m$; (2) Given n, l_m^* is an increasing function of m at a decreasing rate.

Part (1) above says that when the number of bidders in the Dutch auction increases, each of the optimal bid levels will go up albeit the bid decrements will still be increasing as described in Proposition 1. Part (2) reveals the relationship between the highest optimal bid level and the number of bid levels.

Proposition 3: (1) Given m, Z^* is an increasing function of n; (2) Given n, Z^* is an increasing function of m at a decreasing rate.

We see from Part (1) above that the maximum expected revenue increases with the number of auction participants, and Part (2) states that the maximum expected revenue also increases with the number of bid levels.

V. Conclusion

The main objective of this paper is to determine the bid levels to be set to maximize the auctioneer's expected revenue in a Dutch auction given the number of bidders along with the distribution of their valuations. We also seek to characterize the optimal bid levels and the maximum expected revenue. Obviously, more work is required before we gain a fuller understanding of the type of Dutch auctions considered in this study.

For instance, it should be of interest to find out what the results will be if the bidders' valuations are subject to, say, an exponential distribution [2] [5]. Also, there appears to be a growing interest in Dutch auctions with a reserve price [1] or a random number of bidders [3]. These and other issues will be on our research agenda in the future.

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